Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 2, 2002*
Accepted: *November 28, 2002*

Electromagnetic strings: complemetarity between time and temperature

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Abstract: We investigate some of the intricate features in a gravity decoupling limit of a open bosonic string theory, in a constant electromagnetic (EM-) field. We explain the subtle nature of space-time at short distances, due to its entanglement with gauge field windings in the theory. Incorporating the mass-shell condition, we show that the time coordinate is small, of the order of EM-string scale, and the space coordinates are large. We perform a careful analysis in the critical regime to describe the decoupling of a series of gauge-string windings in successions, just below the Hagedorn temperature. We argue for the condensation of gauge-string at the Hagedorn temperature, which is followed by the decoupling of tachyonic particles. We demonstrate the phenomena by revoking the effective noncommutative dynamics for $D(3)$-brane and obtain non-linear corrections to U(1) gauge theory. We discuss the spontaneous breaking of noncommutative U(1) symmetry and show that the Hagedorn phase is described by the non-interacting gauge particles. The notion of time reappears in the phase at the expense of temperature. It suggests a complementarity between two distinct notions, time and temperature, at short distances.

Keywords: Bosonic Strings, D-branes, Non-Commutative Geometry, Thermal Field Theory.

JHEP11(2002)052

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1. Introduction

Conventional notion of space-time becomes subtle, at short distances, due to the existence of intrinsic bounds in string theory. There, the space-time windings grow exponentially and may invalidate the riemannian manifold. Thus, it requires a new setting to describe the physical phenomena at short distances. For instance, an equilibrium thermal string description encounters a bound due to the Hagedorn temperature in the theory. Apart from this, there are other stringy bounds in the theory. Recent idea of Dirichlet (D-) branes in string theory favours the assertion and allows one to probe shorter distances than the string scale [1]. The presence of electromagnetic (EM-) field in string theory [2, 3] further provides evidence for a bound and allows one to access even a smaller distance. However, the minimal length scale in the theory is bounded[1] by a critical value of electric (E-) field [4]–[7]. Under most of the circumstances,[2] the magnetic (B-) field can also be seen to be bounded by the value of the critical E-field in the theory. On one hand, an E-field introduces the notion of temperature and on the other, it initiates small distance phenomena, leading to space-time noncommutativity in string dynamics [8]. In this work,

[1] A bound on electric field, eliminates the tachyons, otherwise present in the (super) string theory.

[2] Except for the perpendicular E- and B-fields.

we focus on these two alternate descriptions, in a open bosonic string theory and find a complementarity between time and temperature. For a comprehensive review on the aspects of strings in EM-field, we refer the reader to ref. [9].

In the recent past, the notion of noncommutative space-time is established in a open string theory, at its boundary, due to the presence of a constant two-form $b_{\mu\nu}$ in the theory. Since the global mode $b_{\mu\nu}$ can not be gauged away by any gauge transformations, the noncommutativity among the space-time coordinates becomes inevitable in the theory. For instance a strong B-field, in absence of an E-field, decouples all the stringy modes from the theory [10]–[13]. In an effective description, it corresponds to a noncommutative U(1) theory, which is perturbatively unitary [14]. However in presence of a non-zero E-field and vanishing B-field, some of the light stringy modes do not decouple from the theory and in the effective description, it corresponds to a noncommutative open string theory (NCOS), instead of a field theory [15]–[21]. In the context, it has been realized that theories with temporal noncommutativity are consistent quantum mechanically. Nevertheless, it is certain that the space-time noncommutative field theories do not possess a unitary S-matrix and lead to acausal behaviour. It means, a stable field theoretic limit does not exist in a space-time noncommutative string theory. In other words, the low energy domain is described by a critical E-field, which can be seen to be necessary to get rid of the instabilities even at the classical level.[3]

In an alternate, thermal, description for the effective NCOS, a bound on E-field can be seen to introduce Hagedorn temperature. In fact, the notion of finite temperature becomes relevant as soon as an E-field is introduced in the theory [3]. At this point, it is inspiring to recall the analogy between Hagedorn temperature in string theory and the deconfining transition in QCD [22, 23]. As a consequence, the study of high temperature string phase is believed to describe a spontaneously broken non-abelian gauge symmetry, which in turn leads to deconfinement in QCD [24]. On the other hand, in a bosonic string theory, it was shown that some of the winding modes become mass-less and describes a phase transition at the Hagedorn temperature [25]. Subsequently, a first order phase transition has been confirmed at Hagedorn temperature, where the mass-less mode behaves as the order parameter in the theory [26]. Thus, the Hagedorn behaviour is intertwined with NCOS, due to the inherent notion of temperature in the theory. Recent investigations [27]–[29] for Hagedorn transition in NCOS theory has been shown to be second order, due to the absence of mass-less closed string modes.

In this paper, we investigate some of the features entangled with the global modes of a two-form b-field,[4] in a open bosonic string theory, at its gravity decoupling limit [16]. We begin with a $(3+1)$-dimensional space-time, describing a $D(3)$-brane at the string world-sheet boundary[5] The string mode expansion in presence of a constant EM-field $(2\pi\alpha' F + b)$

[3]In a quantum picture, the electric field generates charged string pairs from the vacuum, which leads to enormous energy and hence instability in the theory [5].

[4]We recall that an arbitrary b-field is an integral part of the mass-less closed string backgrounds. It combines with the U(1) field strength $F = dA$, present at the string boundary, to retain the gauge invariance in the theory.

[5]An arbitrary dimensional space-time generalization is straight-forward. For N number of $D(3)$-branes,

is reviewed. We perform a careful analysis for the light windings, after decoupling all the mass-less closed string modes from the theory. In the limit $E \to E_c$ and $\alpha' \to 0$, the mass-less closed string modes decouple from the open string and leaves behind a non-gravitational, "rigid" electric string with string tension $T \to \infty$.[6] Alternately, in an effective description, the theory is known to possess noncommutativity among its space and time coordinates and becomes tension-less $T_{\rm eff} \to 0$ [7]. At a critical electric field (E_c), some of the light mass string states remain in the effective NCOS theory and describes the dynamics for "long" strings there. When a B-field is introduced parallel to E-field, the bound on the E-field becomes a bound on B-field as well. In this case, an E-string may be generalized to the EM-string. Since under a strong-week coupling duality, the fields get interchange $E \leftrightarrow B$, the relative field strength becomes unimportant for the case $E \parallel B$, which is unlike to $E \perp B$. It implies that the orientation of B-field with respect to E-field plays an important role in the gravity decoupling limit.

On the other hand, the difference between EM-string theory and that of the fundamental string is in their respective length scales associated with independent sources in the theory. Needless to mention that the prime source to the fundamental string scale is gravity and that to an effective noncommutative string can be seen to be massive gauge theory. In addition, the noncommutative string is defined with a minimal length scale, very unlike to that in a fundamental string. It leads to a conceptual difference between the notion of uncertainty among the space-time coordinates in string theory [30] and in a noncommutative string theory [8]–[21]. The minimal noncommutative string scale does not allow an independent field theoretic description in the theory, instead describes a NCOS theory. It implies that a space-time noncommutative field theory may not be obtained from the NCOS theory. Nevertheless, a field theoretic domain is consistent within a NCOS theory. In this context, we analyze the NCOS modes to explain the cancellation of unphysical states present, otherwise, in a field theoretic domain.

We encounter a mixing of time with space components from the non-zero momentum modes. Incorporating the mass-shell condition in the theory, we show that the time-like dimension is small and argue for the presence of large space dimensions in a $(3+1)$-dimensional NCOS theory. The tiny time-like winding modes give rise to a time translation symmetry and introduces the notion of finite temperature in NCOS theory. We consider the phase transitions associated with the decoupling of mass-less and some of the non-zero mass string modes and obtain an expression for the Hagedorn temperature in the theory. We argue for the condensation of EM-string at Hagedorn temperature. Subsequently, the light modes become tachyonic and decouple from the theory to describe the Hagedorn phase. We present a brief analysis on a space-time noncommutative U(1) theory, in the critical regime. The spontaneous breaking of noncommutative U(1) symmetry [12, 31] is discussed to comment on the Hagedorn phase in the theory.

the gauge group at the string boundary becomes U(N).

[6] E-field in the theory introduces a stretch parallel to the string, where as a B-field gives a stretch along transverse direction to propagation. A priori, a stretch due to B-field gives rise to a width, unlike to tension due to the E-field, since a B-field does no work. However, a B-field is a source for space-like windings and contributes to energy in the theory.

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The paper is organized as follows. In section 2.1, we describe the $D(3)$-brane dynamics in presence of a constant EM-field to make our presentation self-contained. In section 2.2, we obtain a bound on E-field, by briefly reviewing the $D(3)$-brane dynamics described by the Dirac-Born-Infeld (DBI) action and its noncommutative counter-part in the effective description. In section 2.3, we demonstrate the importance of string modes to unitarity in a space-time noncommutative theory and obtain the hamiltonian for the propagation of NCOS. We describe in some detail on the nature of time and space coordinates in section 3.1 for $E \perp B$ and in section 3.2 for $E \parallel B$. In section 3.3, we confirm the mixing of time with space components from the non-zero Fourier modes. In section 4.1, we describe the thermal modes and obtain an expression for the Hagedorn temperature in the theory. In section 4.2, we describe the phase transitions, just below the Hagedorn temperature and account for a time-like noncommutative U(1) theory. In section 4.3, we elaborate on the Hagedorn phase in the context and its similarities with the deconfining phase in QCD. We summarize our results and out-line on some of the further directions in section 5.

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2. Strings in constant electromagnetic field

2.1 Set up: D(3)-brane dynamics

Consider a $D(3)$-brane in a open bosonic string theory. The $D(3)$-brane world-volume dynamics can essentially be described by a open string at its boundary. Since a $D(3)$-brane describes a (3+1)-dimensional space-time X^μ ($\mu = 0, 1, 2, 3$) embedded in string theory, the transverse space to the brane is described by Dirichlet boundary conditions $\delta X^{\perp} = 0$. The $D(3)$-brane effective dynamics can be obtained from the sigma model action for the open string, in a conformal gauge, with $\delta X^{\perp} = 0$. For constant closed string backgrounds: metric $g_{\mu\nu} = g\delta_{\mu\nu}$, two-form $b_{\mu\nu} = b\epsilon_{\mu\nu}$, dilaton and a constant U(1) field strength $F_{\mu\nu}$ at the open string boundary, the action [2] for $D(3)$-brane becomes

$$S = -\frac{1}{4\pi\alpha'}\left[\int_{\Sigma} d\tau d\sigma \partial_a X^\mu \partial^a X^\nu g_{\mu\nu} - \bar{\mathcal{F}}_{\mu\nu}\int_{\partial\Sigma} d\tau \ X^\mu \partial_\tau X^\nu\right] . \tag{2.1}$$

$\bar{\mathcal{F}}_{\mu\nu} = (2\pi\alpha')\mathcal{F}_{\mu\nu} = (2\pi\alpha' F + b)_{\mu\nu}$ denotes the U(1) gauge invariant field strength. The inherent ingredients in $\bar{\mathcal{F}}_{\mu\nu}$ are the gauge field $A_\mu(X)$ and the global two-form $b_{\mu\nu}$. The latter is closed but not an exact form. Since a local transformation can not gauge away the global field, it remains in the theory and can be seen to spontaneously break the U(1) gauge symmetry in the theory. As a result, the $D(3)$-brane world-volume dynamics is described by a noncommutative U(1) symmetry [12].

The string coordinates X^μ satisfies the world-sheet equation of motion $\Box X^\mu = 0$ in bulk, as well as the mixed boundary condition on the boundary[7]

$$g_{\mu\nu}\partial_\sigma X^\nu - \bar{\mathcal{F}}_{\mu\nu}\partial_\tau X^\nu = 0 \, , \tag{2.2}$$

[7]For an infinite strip world-sheet topology, the boundary is described by $\sigma = (0, \pi)$. In fact, the boundary possesses stringy features and only to leading order in α', the boundary describes points there.

where $\bar{\mathcal{F}}_{\mu\nu} = \bar{\mathcal{F}}\epsilon_{\mu\nu}$ is an invertible matrix. The string mode expansion for its coordinates become

$$X^\mu(\tau,\sigma) = x^\mu + 2\alpha' p^\mu \tau + \frac{1}{\pi}(\mathcal{F}^{-1})^{\mu\nu} p_\nu \sigma + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \left(i\alpha_n^\mu \cos n\sigma + \bar{\mathcal{F}}^\mu_\nu \alpha_n^\nu \sin n\sigma \right). \tag{2.3}$$

The mode expansion for the string, contains two independent perturbation parameters (α' and $\mathcal{F}$). However on the $D(3)$-brane world-volume, the effective dynamics is described by a single effective parameter α'_{eff} [12]. We use the notations $\mathcal{F}_{0i} = E_i$ for electric components and $\mathcal{F}_{ij} = \epsilon_{ijk} B_k$ $(i = 1,2,3)$ for magnetic ones in the theory. Explicitly, the EM-field strength can be expressed

$$\mathcal{F}_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}. \tag{2.4}$$

The Poincare dual ${}^\star\mathcal{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\rho\sigma}$ can be expressed

$${}^\star\mathcal{F}^{\mu\nu} = \begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & -E_3 & E_2 \\ -B_2 & E_3 & 0 & -E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}. \tag{2.5}$$

The Lorentz invariants on the $D(3)$-brane world-volume are given by

$${}^\star\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} = -4\,(E\cdot B) \qquad \text{and} \qquad \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} = -2\left(E^2 - B^2\right). \tag{2.6}$$

Since the direction of E-field is all along the string, an electric stretch is parallel (or anti-parallel) to that of string. The stretch builds up an induced (electric) string tension and hence a length scale, smaller in magnitude,[8] to that of the string scale α' in the theory. As a consequence, the tension of the fundamental string receives a correction from that of an E-field. A priori, the effective string tension can be described by

$$T^2_{\text{eff}} = T^2 \pm E^2\,. \tag{2.7}$$

The electric string alone gives rise to a new stringy description, coupled to the conventional string and in a boundary description it corresponds to NCOS.

On the other hand, the B-field induces a "stretch" along the transverse direction to string and sets up a magnetic scale (independent of electric one) in the theory. Needless to mention that the magnetic scale does not contribute to string tension, since no work is done by a B-field alone. Nevertheless, the B-field contributes substantially to energy along a transverse direction to propagation. As a result, the magnitude of T_{eff} (2.7) is scaled by an overall constant factor due to the transverse growth.

[8] Since the noncommutative scale is a short-distance probe for testing string scale.

2.2 Bound on electric (magnetic) field

The fact that space-like noncommutative field theories are perturbatively unitary at one loop level [14] and the time-like ones are not [17], confirms different roles played by the $E-$ and B-fields. We recall [8]–[13] the effective noncommutative $[X^\mu, X^\nu] = i\Theta^{\mu\nu}$ description on the world-volume of a $D(3)$-brane. There the momentum square can be given by the inner product

$$P_{\rm nc} \cdot P_{\rm nc} = -p_\lambda \Theta^{\lambda\mu} \Theta_{\mu\rho} p^\rho \,. \tag{2.8}$$

The constant matrix $\Theta^{\mu\nu}$ can be written in block-diagonal form, without loss of generality[9]

$$\Theta^{\mu\nu} = \begin{pmatrix} 0 & \Theta_E & 0 & 0 \\ -\Theta_E & 0 & 0 & 0 \\ 0 & 0 & 0 & \Theta_B \\ 0 & 0 & -\Theta_B & 0 \end{pmatrix} . \tag{2.9}$$

Simplification in terms of electric Θ_E and magnetic Θ_B parameters yield

$$P^2_{\rm nc} = \Theta_E \left(p_0^2 - p_1^2\right) + \Theta_B \left(p_2^2 + p_3^2\right) . \tag{2.10}$$

For time-like noncommutativity (i.e. $\Theta_E \neq 0$), $P^2_{\rm nc}$ can be negative. The presence of unphysical states lead to a description with non-unitary S-matrix.[10] However a bound on E-field is known to resolve the problem of unitarity in the theory at the expense of the field theoretic dynamics. The precise bound on E-field may be obtained by confining to a physical domain for the brane dynamics, which is described by DBI lagrangian density. For a $D(3)$-brane, it can be simplified to yield

$$\mathcal{L}_{\rm DBI} = -T_D \left[g^4 + g^2 \left(2\pi\alpha'\right)^2 \left(B^2 - E^2\right) - \left(2\pi\alpha'\right)^4 \left(E \cdot B\right)^2 \right]^{1/2} . \tag{2.11}$$

Here $T_D = 1/[g_s(2\pi)^3 \alpha'^2]$ denotes the $D(3)$-brane tension, when g_s corresponds to the closed string coupling. In the weak coupling limit $g_s \ll 1$, the $D(3)$-brane becomes heavy and signifies a smaller probe than the fundamental string. It is important to note that the physical domain in Minkowski space-time is represented by a positive and definite quantity under the square root in eq. (2.11). In the case, when both the Lorentz invariants (2.6) are set to zero, the DBI lagrangian density confirms no bound on E-field. On the other hand, when one or both of the invariants are non-zero, one obtains an upper bound on E-field. For instance when $E \perp B$, the bound on E-field is less restrictive. There, the relative strength between E- and B-fields decides on the cut-off value. It is interesting to note that the bound on E-field takes an unique value for all cases. The critical value can be obtained from eq. (2.11)

$$E_c = \frac{g}{(2\pi\alpha')} \,. \tag{2.12}$$

[9]The block-diagonal form for the noncommutative matrix parameter is obtained with euclidean signatures and an analytic continuation is performed to obtain $\Theta^{\mu\nu}$ in terms of electric Θ_E and magnetic Θ_B parameters.

[10]For space-like noncommutativity (i.e. $\Theta_B \neq 0$, $\Theta_E = 0$), the theory describes physical states, $P^2_{\rm nc} > 0$.

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It corresponds to that of string tension T in the theory. There is no natural bound on B-field, though under certain circumstances, $|B| < |E|$, B-field is bounded. As E increases, the length scale for the electric string also increases. At E_c, it becomes maximum and is equal to that of the fundamental string. Since all the closed string modes decouple in NCOS limit [15, 16], the residual open string becomes rigid, i.e. $T \to \infty$. However, the effective string becomes tension-less $T_{\rm eff} \to 0$. In other words, an E-field drains off the string tension for a (maximal) stable configuration. Than the effective string tension (2.7) can precisely be described by

$$\frac{\alpha'}{\alpha'_{\rm eff}} = \left(g^2 - |2\pi\alpha' E|^2\right)^{\frac{1}{2}} . \tag{2.13}$$

In the limit $E \to E_c$, the relative strength between the length scales becomes $\alpha'/\alpha'_{\rm eff} \to 0$, when $\alpha'_{\rm eff}$ and the effective open string coupling are kept fixed. Though the closed string coupling $g_s = (\alpha'_{\rm eff}/\alpha')^{1/2} \to \infty$, it remains in the bulk and decouples from the string boundary, i.e. from the $D(3)$-brane world-volume dynamics.

In this context, it is useful to recall that a $D(3)$-brane forms a bound state [32, 7] with the fundamental string in presence of an E-field. The effective tension for the bound state can be described by

$$T^2_{\rm eff} = T^2_D + \tilde{E}^2 . \tag{2.14}$$

In the decoupling limit $T_{\rm eff} \to 0$, the critical electric field can be identified with the $D(3)$-brane tension, $\tilde{E}_c = iT_D$. Since the electric field and the $D(3)$-brane tension are real, the DBI dynamics makes sense only when the time-like coordinate is analytically continued. Than, in the limit $\tilde{E} \to \tilde{E}_c$, the closed string modes are cancelled by the electric string and at the same time, the number of "rigid" strings becomes large to yield "long" string in the theory.

2.3 Unitarity

In this section, we comment on unitarity in a space-time noncommutative gauge theoretic regime in a NCOS theory. We obtain the hamiltonian in the theory by taking into account the longitudinal and transverse momenta. Some of the light string modes become prominent in the theory and can be seen to modify the hamiltonian.

Now consider parallel E and B fields in a open bosonic string theory. The total momentum $\mathcal{P}$ for the string in presence of EM-field can be expressed in terms of the momenta, along the direction of motion $P^{||}_\mu(\tau,\sigma)$ and along the transverse direction $P^{\perp}_\mu(\tau,\sigma)$. It becomes

$$\mathcal{P}_\mu = \left[\int_0^\pi d\sigma P^l_\mu\right]_{\tau=0} + \left[\int_0^\infty d\tau P^\perp_\mu\right]_{\sigma=0} . \tag{2.15}$$

The longitudinal and transverse momenta can be obtained from the lagrangian density in eq. (2.1). They are

$$P^l_\mu = \frac{1}{2\pi\alpha'}\left(g_{\mu\nu}\partial_\tau X^\nu + \bar{\mathcal{F}}_{\mu\nu}\partial_\sigma X^\nu\right) \qquad \text{and} \qquad P^\perp_\mu = -\frac{1}{2\pi\alpha'}\left(g_{\mu\nu}\partial_\sigma X^\nu + \bar{\mathcal{F}}_{\mu\nu}\partial_\tau X^\nu\right) . \tag{2.16}$$

The total longitudinal momenta can be obtained by integrating over the infinitesimal flow. The longitudinal momenta consist of a component $\mathcal{P}^l_0$ along the propagation and $\mathcal{P}^l_1$ along the E-field. The remaining two components $(\mathcal{P}^l_2, \mathcal{P}^l_3)$ are transverse to $(\mathcal{P}^l_0, \mathcal{P}^l_1)$. Explicitly, the momenta become

$$\begin{aligned}
\mathcal{P}^l_0 &= -\left(g - g^{-1}\bar{E}^2\right) p^0 + i(4E)\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{(2n+1)}\alpha^1_{2n+1}\,,\\
\mathcal{P}^l_1 &= \left(g - g^{-1}\bar{E}^2\right) p^1 - i(4E)\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{(2n+1)}\alpha^0_{2n+1}\,,\\
\mathcal{P}^l_2 &= \left(g - g^{-1}\bar{B}^2\right) p^2 + i(4B)\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{(2n+1)}\alpha^3_{2n+1}\,,\\
\mathcal{P}^l_3 &= \left(g - g^{-1}\bar{B}^2\right) p^3 - i(4B)\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{(2n+1)}\alpha^2_{2n+1}\,.
\end{aligned} \tag{2.17}$$

The corrections to constant momentum modes are associated with the EM-field. They are the constant and non-zero momentum corrections and as a whole describes a complex longitudinal momenta. Intuitively, the longitudinal momenta describes a space-time non-commutative "field" theoretic regime in the string theory. The complex momenta (2.17) in the regime, describes negative norm states in addition to the physical states and lead to a non-unitary description. Nevertheless, there are transverse flow of momenta in the theory due to the stringy description. It can be checked that the transverse momenta $\mathcal{P}^{\perp}_{\mu}$, cancel the imaginary non-zero modes in longitudinal momenta. The total momentum $\mathcal{P} = \mathcal{P}^l + \mathcal{P}^{\perp}$ becomes proportional to that of the constant momentum modes (2.17). The proportionality constant truns out to be the modified metric $g_{\mu\nu}\rightarrow(g_{\mu\nu} - \bar{\mathcal{F}}_{\mu\lambda}g^{\lambda\rho}\bar{\mathcal{F}}_{\rho\nu})$ in the NCOS theory. Thus the EM-string theory describes the propagation of a non-gravitating "rigid" string. Interestingly, the absence of closed string poles have been confirmed in the non-planar one-loop diagram in NCOS theory [16]. Though the presence of graviton pole in the planar one-loop diagram is essential to retain the unitarity in open string theory, a NCOS theory does not require so.

Now the hamiltonian describing the propagation of string in presence of EM-field can be given

$$H = \int_0^{\pi} d\sigma \left(P^{||}_{\mu}\partial_{\tau}X^{\mu} + P^{\perp}_{\mu}\partial_{\sigma}X^{\mu} - \mathcal{L}\right), \tag{2.18}$$

where the lagrangian $\mathcal{L}$ can be obtained from the sigma model action (2.1). The hamiltonian ($H = L^b_0$, zero-mode for Virasoro generators in presence of a constant b-field) becomes

$$L^b_0 = \alpha' p^2 - \alpha'\left(\bar{\mathcal{F}}g^{-1}\bar{\mathcal{F}}\right)_{\mu\nu} p^{\mu}p^{\nu} + N_{\rm osc}\,, \tag{2.19}$$

where $N_{\rm osc}$ denotes the number operator and its eigen values are $(0, 1, 2, \ldots)$ in the theory. The hamiltonian receives a correction due to the light string modes in the theory. Then the mass-shell condition becomes

$$\left(L^b_0 - 1\right)|\mathrm{phy}\rangle = 0\,. \tag{2.20}$$

The poles in the theory describe physical as well as unphysical states. However the unphysical states are orthogonal to the physical ones. The orthogonality condition can be implemented to get rid of the unphysical states from the NCOS theory. We postpone the relevance of mass-shell condition to sections 3 and 4.

3. New string modes: Small time-like dimension

3.1 $E \perp B$

Consider an EM-field, with $E = (E_1, 0, 0)$ and $B = (0, B_2, 0)$, compatible with perpendicular electric and magnetic field configurations $E \cdot B = 0$. Any other combination for the field components satisfying $E \cdot B = 0$, may be obtained from the above case. For instance, if one switches on E_3 (or B_3) in addition to E_1 and B_2, the electric (magnetic) components can be rotated to yield a single independent component.

The boundary conditions (2.2) for the perpendicular fields become

$$\begin{aligned} \partial_\sigma X^0 - \frac{E}{E_c}\partial_\tau X^1 &= 0\,, \\ \partial_\sigma X^1 + \frac{E}{E_c}\partial_\tau X^0 + \frac{B}{E_c}\partial_\tau X^3 &= 0\,, \\ \partial_\sigma X^2 &= 0 \\ \text{and} \quad \partial_\sigma X^3 - \frac{B}{E_c}\partial_\tau X^1 &= 0\,. \end{aligned} \tag{3.1}$$

The mode expansion (2.3) can be expressed explicitly

$$\begin{aligned} X^0 &= x^0 + 2\alpha' p^0 \tau + \frac{1}{\pi} g E^{-1} p^1 \sigma + \\ &\quad + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \left(i\alpha_n^0 \cos n\sigma + g\bar{E}^{-1} \alpha_n^1 \sin n\sigma \right), \\ X^1 &= x^1 + 2\alpha' p^1 \tau + \frac{1}{\pi} \left(g E^{-1} p^0 - g B^{-1}\, p^3 \right) \sigma + \\ &\quad + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \left(i\alpha_n^1 \cos n\sigma + \left(g\bar{E}^{-1}\alpha_n^0 - g\bar{B}^{-1}\alpha_n^3 \right) \sin n\sigma \right), \\ X^2 &= x^2 + 2\alpha' p^2 \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \alpha_n^2 \cos n\sigma \\ \text{and} \quad X^3 &= x^3 + 2\alpha' p^3 \tau + \frac{1}{\pi} g B^{-1} p^1 \sigma + \\ &\quad + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \left(i\alpha_n^3 \cos n\sigma + g\bar{B}^{-1}\ \alpha_n^1\ \sin n\sigma \right). \end{aligned} \tag{3.2}$$

The new string modes in the spectrum are entirely due to the EM-field and give rise to a transverse growth and may be interpreted as the additional winding modes in the theory. They possess a small mass and consist of temporal and spatial windings. Apart from them, there are closed string (mass-less) winding modes in the theory.

In the case, a variation in E-field is independent from that in B-field and vice-versa. However, the relative strength between the fields decides over the fate of the gravity decoupled theory. There are three distinct cases: i).$|E| = |B|$, ii). $|E| < |B|$ and iii). $|E| > |B|$. For an equal field strength, eq. (2.11) confirms no bound and hence a string decoupling limit (large E and large B) can be reached. It corresponds to a light-like noncommutative

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field theory [33]. For the second case ($|E| < |B|$), there is no bound on the fields, which in turn leads to a space-like noncommutative field theory in the string decoupling limit. Unlike the (ordinary) field theory, the noncommutative one possesses winding modes and shares some of the properties of a thermal field theory [34]. In this case, the E-field is not bounded by the string tension rather it is bounded by the B-field strength. For a strong B-field (large B limit), E-field can also take a large value. The boundary conditions (3.1) to leading order in B-field yields field theoretic modes

$$X^1 \to x^1 \quad \text{and} \quad X^3 \to x^3 \,. \tag{3.3}$$

Since some of the magnetic field components ($B_1 = 0$ and $B_3 = 0$) are zero, the complete gravity decoupling limit for $E \perp B$ becomes $\alpha' \to 0$ and large B (when α' is kept fixed). For $\alpha' \to 0$, all the oscillator modes drop out from the spectrum including the ones from X^0 and X^2

$$X^0 \to x^0 \quad \text{and} \quad X^2 \to x^2 \,. \tag{3.4}$$

In the third case $|E| > |B|$, due to a bound on E-field (2.12), the string decoupling limit is replaced by that of gravity and describes a time-like noncommutative theory. Though a variation in E-field does not affect the B-field, in the decoupling limit, the magnitude of magnetic scale is bounded by that of electric, which in turn coincides with the intrinsic scale in the theory. Then the gravity decoupling limit is described by

$$gE^{-1} \to 2\pi\alpha' \quad \text{and} \quad gB^{-1} \to 2\pi\alpha'_{\perp} \,, \tag{3.5}$$

where α' and $\alpha'_{\perp}$, respectively, denote the "rigid" scales for electric string and transverse (magnetic) width in the theory. Interestingly, the "rigid" electric (magnetic) string can be identified with the electric (magnetic) dipole there. The EM-string modes in the limit can be obtained from eq. (2.3). They are

$$\begin{aligned}
X^0 &= x^0 + 2\alpha' p^0 \tau + 2\alpha' p^1 \sigma + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \left(i\alpha_n^0 \cos n\sigma + \alpha_n^1 \sin n\sigma \right) , \\
X^1 &= x^1 + 2\alpha' p^1 \tau - 2\alpha' p^0 \sigma + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \left(i\alpha_n^1 \cos n\sigma - \alpha_n^0 \sin n\sigma \right) + \\
&\quad + 2\alpha'_{\perp} p^3 \sigma - \sqrt{2\alpha'_{\perp}} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \alpha_n^3 \sin n\sigma \,, \\
X^2 &= x^2 + 2\alpha' p^2 \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \alpha_n^2 \cos n\sigma \\
\text{and} \quad X^3 &= x^3 + 2\alpha' p^3 \tau + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \alpha_n^3 \cos n\sigma + \\
&\quad + 2\alpha'_{\perp} p^1 \sigma + \sqrt{2\alpha'_{\perp}} \sum_{n\neq 0} \frac{e^{-in\tau}}{n} \alpha_n^1 \sin n\sigma \,.
\end{aligned} \tag{3.6}$$

The length scales $\sqrt{2\alpha'} = l_s$ and $\sqrt{2\alpha'_{\perp}} = l_{\perp}$ are along the direction of E- and B-fields ($l_s < l_{\perp}$) and they signify string and magnetic scales, respectively, in the theory. The presence of winding modes (for X^0, X^1 and X^3) in a gravity decoupled theory may be surprising at first sight. However, as mentioned in section 2.3., the winding modes in EM-string theory are entirely due to the (critical) EM-field and possess a small mass. The presence of momentum as well as winding modes is unlike to that in a conventional string. Nevertheless, their simultaneous presence is justifiable due to the noncommutative constraint in the effective theory.

Different scales l_s and $l_\perp$ confirm two independent winding modes for X^1 with respective radii R and $R_\perp$. Identification of the EM-string winding modes in eq. (3.6) under T-duality symmetry, interrelates the winding radii. Then the string modes become

$$\begin{aligned} & X^0 \rightarrow X^0 + m\Big(2\pi R\Big)\ , \\ & X^1 \rightarrow X^1 + n\Big(2\pi R^d\Big) + n'\Big(2\pi \frac{\alpha'}{\alpha'_\perp} R^d_\perp\Big)\ , \\ & X^2 \rightarrow X^2 \\ \text{and} \quad & X^3 \rightarrow X^3 + w\Big(2\pi \frac{\alpha'}{\alpha'_\perp} R_\perp\Big)\ , \end{aligned} \tag{3.7}$$

where (m, n, n', w) are arbitrary integers and denote the respective winding numbers for the EM-string coordinates. The dual winding radii are $R^d = \alpha'/R$ and $R^d_\perp = \alpha'/R_\perp$. Though the winding radius for X^3 is $R_3 = (\alpha'/\alpha'_\perp)R_\perp$, its ratio with R^d_3 is independent of the string scale $R_\perp/R^d_\perp$. The effective winding radius for X^1 becomes

$$R_1 = \frac{\alpha'}{R} + \left(\frac{\alpha'^2}{\alpha'_\perp}\right)\frac{1}{R_\perp}\ . \tag{3.8}$$

Since $E > B$ implies $(\alpha'^2/\alpha'_\perp) \ll 1$, the winding radius for X^1 becomes equal to the T-dual of X^0. For a large winding radius, the corresponding string coordinate describes a soliton string state. Its T-dual coordinate becomes small and describes a particle state there.

The most revealing feature of the winding mode analysis is the fact that the time-like (along with some of the space-like) coordinate becomes compact in an alternate picture to that with lorentzian space-time signature. In fact the eq.(14) in the gravity decoupling limit confirms the presence of tachyonic states in the $D(3)$-brane spectrum with lorentzian signature of space and time. In an equivalent picture, one may view the brane spectrum as a physical one at the expense of imaginary E-field, which in turn implies a real E-field in euclidean time. As a consequence, the notion of temperature ($\mathcal{T}$) becomes prominent, in the decoupling limit. In other words, the notion of time breaks down in the limit, leading to imaginary time, which in turn gives rise to the notion of temperature in the theory.

Since the U(1) field strength is a constant, the gauge degrees of freedom are intertwined with that of the string $A_\mu(X) = -\frac{1}{2}\mathcal{F}_{\mu\nu}X^\nu$. The light modes in gauge field introduces the notion of "gauge-string" in the theory. They may be obtained from eq. (3.7). It is given by

$$\begin{aligned} & A_0 \rightarrow A_0 - \left(\frac{\pi E}{\alpha'}\right)\frac{1}{R}\ , \\ & A_1 \rightarrow A_1 + (\pi E)\, R - \left(\pi\alpha'_\perp B\right)\frac{1}{R_\perp}\ , \end{aligned}$$

$$
\begin{aligned}
&A_2 \to A_2 \\
\text{and} \qquad &A_3 \to A_3 - (\pi B)\, R_\perp \,..
\end{aligned}
\tag{3.9}
$$

The gauge field windings imply a quantization of its conjugate momenta, i.e. the E-field is quantized[11] and effectively results in quantization of the string slope $\alpha'_{\rm eff} = n\alpha'$, where α' is the minimal scale. Intuitively, the tension-less effective string at E_c is made from a large number of "rigid" EM-strings joined end to end and are aligned along the E-field. Since the E-field is along a compact X^1, the effective open string becomes oriented along a closed loop and apparently resembles to that of a dummy closed string.

To obtain a bound on the time-like winding radius in the EM-string theory, consider the hamiltonian (2.19) for $E \perp B$. It can be written as

$$
L_0^b = L_0 + \left(\alpha' + \alpha'_\perp\right) (p_1)^2 + \alpha'_\perp (p_3)^2 \,, \tag{3.10}
$$

where $L_0 \ = \alpha' p^2 + N_{\rm osc}$ is the hamiltonian in absence of light string modes (analogous to a free hamiltonian) in the theory. The mass-shell condition (2.20), can be simplified for a winding state (with winding number ± 1), to yield

$$
R = \sqrt{\alpha'} \,, \quad \text{and} \quad R_\perp = \sqrt{\alpha'_\perp} \,. \tag{3.11}
$$

It implies that the radius for the time-like coordinate X^0 is of order of (electric) string length l_s and that of X^3 is of order of (magnetic) transverse width $l_\perp$. Since $l_s < l_\perp$, the radius for X^0 is smaller than that for X^3. Furthermore, a small time-like dimension in the theory implies that X^1 is large (3.8). Thus, the time-like coordinate possesses a large number of closely spaced tiny intervals $(2\pi R)$, which can be approximated as a continuous (constant) parameter $m(2\pi R) \to C$. As a result, the theory possesses a time-like killing vector $(\partial/\partial X^0)$ and is invariant under a transformation

$$
X^0 {\to} X^0 + C \,. \tag{3.12}
$$

The origin of time translation symmetry is in the E-field and makes sense only at the decoupling limit. The manifestation of time translation can also be understood from T-duality. A priori, it reduces the dimension of space-time from $(3+1)$ to 3-dimensions. However a careful mode analysis implies a reduction in degrees of freedom by one unit in the constant mode sector, which is due to the interchange of winding and momentum modes. It is established that the non-zero momentum modes in the theory, introduce an additional continuous degree of freedom [26], which accounts for the 4th dimension. Eventually, the degrees of freedom in the gauge-string theory remains unchanged.

On the other hand, a close look on the center of mass (cm) for the gauge string coordinates may be useful to explain the origin of continuous degrees of freedom from

[11] For example, the quantization of E-field on a D-string is discussed, in a different context, in our earlier work [35]. In the present case, though the E-field is on a $D(3)$-brane world-volume, it possesses a single component.

the non-zero momentum modes in the theory rather than the constant modes. The cm-coordinates can be obtained for the string modes

$$
\begin{aligned}
x^0_{cm} &= x^0 + \pi\alpha' p^1 + \frac{2}{\pi}\sqrt{2\alpha'}\sum_n \frac{1}{(2n+1)^2}\alpha^1_{(2n+1)}\,, \\
x^1_{cm} &= x^1 + \pi\alpha' p^0 + \frac{2}{\pi}\sqrt{2\alpha'}\sum_n \frac{1}{(2n+1)^2}\alpha^0_{(2n+1)} + \\
&\qquad + \pi\alpha'_\perp p^3 + \frac{2}{\pi}\sqrt{2\alpha'_\perp}\sum_n \frac{1}{(2n+1)^2}\alpha^3_{(2n+1)}\,, \\
x^2_{cm} &= x^2\,, \\
x^3_{cm} &= x^3 - \pi\alpha'_\perp p^1 + \frac{2}{\pi}\sqrt{2\alpha'_\perp}\sum_n \frac{1}{(2n+1)^2}\alpha^1_{(2n+1)}\,.
\end{aligned}
\tag{3.13}
$$

Firstly, the expressions account for the non-locality in time (x^0) and space (x^1, x^3) coordinates, since the cm-coordinates satisfy $[x^\mu_{cm}, x^\nu_{cm}] = 0$. Secondly, the time coordinate mixes with the spatial component of the momentum modes and vice-versa. Intutively, the mixing of time is in the spirit of light-cone coordinates in field theory. However the subtlety is due to the space-time noncommutativity, which is tied with the non-zero modes and hence stringy description comes into picture. In addition, the presence of constant as well as non-zero momentum modes in the expression for cm-coordinates is a new phenomenon in comparison to that of a conventional string. In fact, the non-zero momentum modes are the rarely spaced, constant, light ones from the oscillators in the theory. Since under T-duality, winding and momentum modes are interchanged, the number of degrees of freedom remains unchanged in the theory. The origin of the light states in the cm-coordinates is due to the broken string world-sheet parity. Under such a global discrete symmetry, the string modes become $x^\mu \to x^\mu$, $p_\mu \to p_\mu$ and $\alpha^\mu_n \to (-1)^n \alpha^\mu_n$. It can be checked that under the string world-sheet symmetry all the oscillator modes in $x^\mu_{\rm cm}$ are projected out. However in presence of an E-field, such a symmetry is not respected.

3.2 $E \parallel B$

In this section we consider a parallel field[12] configuration $E = (E_1, 0, 0)$ and $B = (B_1, 0, 0)$. Here a bound on E-field becomes a bound on B-field as well. Since the difference with the perpendicular field configuration is due to the orientation of B-field, the description for the time-like string coordinate X^0 remains identical in both cases. Also, the boundary conditions (2.2) for g_{00} and g_{33} components are the ones, obtained for perpendicular fields in eq. (3.1). The remaining two boundary conditions can be given

$$
\begin{aligned}
&\partial_\sigma X^1 + \frac{E}{E_c}\partial_\tau X^0 = 0 \\
\text{and}\quad &\partial_\sigma X^2 - \frac{B}{E_c}\partial_\tau X^3 = 0\,.
\end{aligned}
\tag{3.14}
$$

[12] The case for anti-parallel electric and magnetic fields can be obtained from the parallel case discussed. In fact, when E- and B-fields are at an angle other than $\pi/2$, the analysis is analogous to that in the parallel case.

The space-like string coordinates can be obtained from eq. (2.3). They are

$$X^1 = x^1 + 2\alpha' p^1\tau - \frac{1}{\pi} gE^{-1}p^0\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(i\alpha_n^1\cos n\sigma + g\bar{E}^{-1}\alpha_n^0\sin n\sigma\right),$$
$$X^2 = x^2 + 2\alpha' p^2\tau + \frac{1}{\pi} gB^{-1}p^3\sigma\sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(i\alpha_n^2\cos n\sigma + g\bar{B}^{-1}\alpha_n^3\sin n\sigma\right),$$
$$X^3 = x^3 + 2\alpha' p^3\tau - \frac{1}{\pi} gB^{-1}p^2\sigma\sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(i\alpha_n^3\cos n\sigma - g\bar{B}^{-1}\alpha_n^2\sin n\sigma\right). \quad (3.15)$$

In the case, all the string coordinates possess light string modes, unlike to that in the perpendicular field configuration. The modes for X^0 and X^3 remain unchanged, from that of $E \perp B$. On the other hand, the light string modes for X^1 in the perpendicular field case is shared by the X^1 and X^2 for $E \parallel B$. In the gravity decoupling limit (3.5), the space-like gauge-string modes become

$$X^1 = x^1 + 2\alpha' p^1\tau - 2\alpha' p^0\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(i\alpha_n^1\cos n\sigma + \alpha_n^0\sin n\sigma\right),$$
$$X^2 = x^2 + 2\alpha' p^2\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha_n^2\cos n\sigma +$$
$$+2\alpha'_\perp p^3\sigma + \sqrt{2\alpha'_\perp}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha_n^3\sin n\sigma,$$
$$X^3 = x^3 + 2\alpha' p^3\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha_n^3\cos n\sigma -$$
$$-2\alpha'_\perp p^2\sigma - \sqrt{2\alpha'_\perp}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha_n^2\sin n\sigma. \quad (3.16)$$

Since the mass-less windings are decoupled in the limit, the remaining light windings are due to the EM-field. There, the winding radius for X^3 is completely independent from that in X^1, (unlike to that in $E \perp B$) rather it is T-dual to that in X^2. Two of the space-like light modes in the theory are

$$X^1 \to X^1 + n\left(2\pi R^d\right) \quad \text{and} \quad X^2 \to X^2 + n'\left(2\pi\frac{\alpha'}{\alpha'_\perp}R_\perp^d\right). \quad (3.17)$$

The modes for X^0 and X^3 remain identical to that in $E \perp B$, obtained in eq. (3.7). The winding radii for the transverse coordinates X^2 and X^3 are dependent on the relative strength $(\alpha'_\perp/\alpha')$. Their ratio is independent of field strengths and can be given by $(R_\perp^d/R_\perp)$. It confirms a "small" dimension to X^3 (or X^2) and a large dimension to X^2 (or X^3) in the transverse space. In addition, there are windings in the gauge field and they can be given by

$$A_0 \to A_0 - \left(\frac{\pi E}{\alpha'}\right)\frac{1}{R},$$

$$\begin{aligned} &A_1 \to A_1 + (\pi E)\, R\,, \\ &A_2 \to A_2 - \left(\pi\alpha'_{\perp} B\right) \frac{1}{R_{\perp}} \\ \text{and} \quad &A_3 \to A_3 - (\pi B)\, R_{\perp}\,. \end{aligned} \tag{3.18}$$

The presence of light string in the gauge field account for the non-locality in gauge theory. As discussed for $E \perp B$, the small radius R in A_1, implies a quantization of E-field and hence an quantized effective string scale $\alpha'_{\rm eff}$.

Now the hamiltonian (2.19) for the gauge-string becomes

$$L_0^b = L_0 + \alpha'(p_1)^2 + \alpha'_{\perp}(p_{\perp})^2\,, \tag{3.19}$$

where $p_{\perp}$ denotes transverse momenta to E-field and to the direction of propagation. The mass-shell condition (2.20) for a winding state (with winding number ± 1) becomes

$$\left(\alpha' - \alpha'_{\perp}\right)(p_1)^2 + \alpha'_{\perp}(p_0)^2 = 1\,. \tag{3.20}$$

For an equal field strength $|E| = |B|$, the mass-shell condition can be further simplified and one obtains a bound on the light-like radius $R = \sqrt{\alpha'}$. As expected, the radius R is precisely the one obtained in eq. (3.11) for $E \perp B$. A priori for $|E| \neq |B|$, the mass-shell condition (3.20) can be seen to introduce a different bound on the radius R. Nevertheless, in the decoupling limit $\alpha'_{\perp} \to \alpha'$, the minimal value for R turns out to be of the order of l_s. The gauge-string scale for time-like intervals in the limit, describes a time translation symmetry (3.12) in the theory. The symmetry is intrinsic to the E-field alone, at its critical value, and is independent of the orientation of B-field in the theory. The non-zero mass space-time windings introduce discrete space-time structure in the theory. However in the limit of critical E-field, the time-like intervals are small enough in comparison to the space-like intervals. In addition, at E_c, it gives rise to a large number of time-like windings. As a result, the time coordinate can be approximated as continuous rather than discrete. However due to the translation symmetry, the notion of time decouples from the theory without affecting the degrees of freedom. In other words, the time coordinate does not retain its identity and mixes with the space components of non-zero momentum modes. At the expense of time, the notion of temperature becomes prominent in the theory.

3.3 Mixing of time with space components

In this section, we describe the interplay of time with space components of the non-zero momentum modes in the gravity decoupling limit. A non-trivial mixing of time with space occurs in a gauge-string description with an arbitrary orientation of B-field with respect to the E-field. However a mixing among the space coordinates (rotation) takes place for non-perpendicular fields. Taking account for a mixing in the momentum modes, separately in constant and in oscillator sector, the time-like coordinate in eq. (3.6) can be re-expressed

$$X^0 = x^0 + 2\alpha' p^- \tau_- + 2\alpha' p^+ \tau_+ + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau_-}}{n}\alpha_n^- + i\sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{-in\tau_+}}{n}\alpha_n^+\,, \tag{3.21}$$

$$\text{where} \quad p^{\pm} = \frac{1}{2}\left(p^0 \pm p^1\right)\,, \quad \alpha_n^{\pm} = \frac{1}{2}\left(\alpha_n^0 \pm a_n^1\right) \quad \text{and} \quad \tau_{\pm} = \tau \pm \sigma\,. \tag{3.22}$$

Similarly, one of the transverse space-like coordinates in eq. (3.16) can be re-expressed

$$X^2 = x^2 + 2\alpha' p^{s-}\tau_- + 2\alpha' p^{s+}\tau_+ + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau_-}}{n}\alpha_n^{s-} + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{e^{-in\tau_+}}{n}\alpha_n^{s+}\,, \tag{3.23}$$

$$\text{where} \quad p^{s\pm} = \frac{1}{2}\left(p^2 \pm \left(\frac{\alpha'_\perp}{\alpha'}\right)p^3\right)\,, \quad \alpha_n^{s\pm} = \frac{1}{2}\left(\alpha_n^2 \pm \left(\frac{\alpha'_\perp}{\alpha'}\right)^{1/2}\alpha_n^3\right)\,. \tag{3.24}$$

The left-right independence of gauge string coordinate allows one to express

$$X(\tau,\sigma) = X_L(\tau_-) + X_R(\tau_+)\,. \tag{3.25}$$

The left and right moving sectors in the open string coordinate is a new observation and is essentially due to the new string modes in the theory. It is the E-field, which in the first place, drains off the closed string from the gauge-string spectrum in such a way that the effective string becomes tension-less. Interestingly, in the process of draining off, the open string becomes "rigid" and a large number of them (as E-field increases) join end to end to form a close loop along the direction of E-field. In the effective description, the "rigid" string configuration describes a tension-less "long" string.

On the other hand, the expressions (3.21)-(3.23), show that a mixing of time and space components, takes away all the string modes in the left and right moving sectors. Then, the zero-mode for the Virasoro generators become

$$\begin{aligned} L_0^- &= \alpha'(p^-)^2 + N_{\rm osc}^-\,, \\ L_0^+ &= \alpha'(p^+)^2 + N_{\rm osc}^+\,, \\ L_0^{s-} &= \alpha'(p^{s-})^2 + N_{\rm osc}^{s-} \\ \text{and} \quad L_0^{s+} &= \alpha'(p^{s+})^2 + N_{\rm osc}^{s+}\,, \end{aligned} \tag{3.26}$$

where $N_{\rm osc}^{\pm}$ and $N_{\rm osc}^{s\pm}$ denote the left and right number operators in the theory. The mass-shell condition (2.20) can be re-expressed as

$$\left(L_0^- + L_0^+ + L_0^{s-} + L_0^{s-} - 1\right)|{\rm phy}\rangle = 0\,. \tag{3.27}$$

For constant modes, the mass-shell condition can be simplified in terms of the original (un-mixed) string coordinates. Then the winding state with mass M, for a single winding, can be given

$$\frac{\alpha'}{2}M^2 = \frac{1}{\alpha'}R^2 - 1\,. \tag{3.28}$$

In the decoupling limit, the on-shell condition introduces a minimum value to the radius $R \geq \sqrt{\alpha'}$ for the time-like coordinate. The lower bound on R is the one obtained in eq. (3.11) for the gauge string without taking account for the mixing of coordinates. In other words, the analysis confirms the mixing of time with space components in the gauge string theory.

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4. Effective thermal string description

4.1 Hagedorn temperature

In the previous section, we noticed that the notion of temperature is genuine in presence of an E-field. Then, the gauge string can be re-interpreted as a thermal string in the theory. In this section, we analyze the gravity decoupling limit and subsequently, the light gauge-string(s) decoupling limit, in an alternate formulation of thermal string theory. Interpreting the decoupling as a phase transition, we investigate for the Hagedorn phase in the theory.

In the gravity decoupling limit, the effective string scale $\alpha'_{\rm eff}$ can be obtained from the eq. (2.13). In the limit $(g-\bar{E})\rightarrow 0^+$ (i.e. $E\rightarrow E_c$ from below), the string scale α' can be replaced by a large scale $\alpha'_{\rm eff}$ and $\alpha'_{\perp}\rightarrow(\alpha'_{\perp})_{\rm eff}$ in the theory. The mode expansion for the gauge-string for $E\perp B$ (3.2) can be re-expressed for a thermal string. It can be given

$$\begin{aligned}
X^0 &= x^0 + \frac{(4\pi\alpha'_{\rm eff})m'}{\beta}\tau + \frac{m\beta}{\pi}\sigma + \sqrt{2\alpha'_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(i\alpha^0_n\cos n\sigma + \alpha^1_n \sin n\sigma\right),\\
X^1 &= x^1 + \frac{(4\pi\alpha'_{\rm eff})n'}{\beta_d}\tau - \frac{n\beta_d}{\pi}\sigma - \frac{q\gamma_d}{\pi}\sigma + \sqrt{2(\alpha'_{\perp})_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha^3_n\sin n\sigma -\\
&\quad -\sqrt{2\alpha'_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(i\alpha^1_n\ \cos n\sigma + \alpha^0_n\ \sin n\sigma\right),\\
X^2 &= x^2 + \frac{(4\pi\alpha'_{\rm eff})q'}{\gamma_d}\tau + i\sqrt{2\alpha'_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha^2_n\ \cos n\sigma\\
\text{and}\quad X^3 &= x^3 + \frac{(4\pi\alpha'_{\rm eff})w'}{\gamma}\tau - \frac{w\gamma}{\pi}\sigma + \sqrt{2(\alpha'_{\perp})_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha^1_n\sin n\sigma +\\
&\quad + i\sqrt{2\alpha'_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha^3_n\ \cos n\sigma\,,
\end{aligned} \tag{4.1}$$

where the thermal parameter $\beta = 2\pi R$ and the associated parameter $\gamma = 2\pi(\alpha'_{\perp}/\alpha')_{\rm eff}R_{\perp}$. Under S-duality, the parameters exchange their role $\beta\leftrightarrow\gamma$ in the theory. The thermal duality relation can be expressed

$$\beta_d = \frac{4\pi^2\alpha'_{\rm eff}}{\beta}\,. \tag{4.2}$$

Similarly, one can also obtain a duality relation involving γ-parameter in the theory. In the closed string decoupling limit, β is small and its dual β_d becomes large and vice-versa. It is in agreement with the observation that the winding radius for X^1 is large.

When the orientation of a B-field is changed by $\pi/2$, i.e. $E\parallel B$, some of the space-like thermal string coordinates (3.16) are different than that of $E\perp B$. They can be re-expressed in terms of thermal string modes

$$X^1 = x^1 + \frac{(4\pi\alpha'_{\rm eff})n'}{\beta_d}\tau - \frac{n\beta_d}{\pi}\sigma - \sqrt{2\alpha'_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(i\alpha^1_n\cos n\sigma + \alpha^0_n\sin n\sigma\right)$$

$$\text{and} \quad X^2 = x^2 + \frac{(4\pi\alpha'_{\rm eff})q'}{\gamma_d}\tau + \frac{q\gamma_d}{\pi}\sigma + i\sqrt{2\alpha'_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha_n^2\cos n\sigma +$$
$$+\sqrt{2(\alpha'_\perp)_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\alpha_n^3\ \sin n\sigma\,. \tag{4.3}$$

The remaining mode expansion for X^0 and X^3, remains identical to that of $E \perp B$ (4.1). The smallness of winding radius R in the gravity decoupling limit gives rise to a high temperature phase in the theory.

In either case ($E \perp B$ or $E \parallel B$), there are four light string modes corresponding to space and time coordinates in the theory. The mode expansion for thermal string confirms that the contribution to winding energy from X^0 (and X^3 for $E \perp B$) is equal to that for the momentum mode in X^1 (and X^2 for $E \perp B$) due to the thermal duality symmetry (4.2). Taking into account the winding energy, the zero mode part of the Virasoro generator for open thermal string becomes

$$L_0^b = -\frac{\alpha'_{\rm eff}}{2}M^2 + \mathcal{N}_{\rm osc} + \frac{\alpha'_{\rm eff}}{2}\left(\frac{2\pi m'}{\beta} + \frac{m\beta}{2\pi\alpha'_{\rm eff}}\right)^2 + \frac{\alpha'_{\rm eff}}{2}\left(\frac{2\pi n'}{\beta_d} + \frac{n\beta_d}{2\pi\alpha'_{\rm eff}}\right)^2 +$$
$$+\frac{\alpha'_{\rm eff}}{2}\left(\frac{2\pi q'}{\gamma} + \frac{q\gamma_d}{2\pi\alpha'_{\rm eff}}\right)^2 + \frac{\alpha'_{\rm eff}}{2}\left(\frac{2\pi w'}{\gamma_d} - \frac{w\gamma_d}{2\pi\alpha'_{\rm eff}}\right)^2, \tag{4.4}$$

where $\mathcal{N}_{\rm osc}$ denotes the number operator. For a pure winding state with a single winding , the mass-shell (2.20) condition yields

$$\frac{\alpha'_{\rm eff}}{2}M^2 = \frac{1}{\alpha'_{\rm eff}}\left(\frac{\beta}{2\pi}\right)^2 - 1\,. \tag{4.5}$$

In the gravity decoupling limit, the free-energy vanishes $M = 0$ and one obtains a critical (Hagedorn) temperature

$$\mathcal{T}_H^{\rm eff} = \frac{1}{2\pi\sqrt{\alpha'_{\rm eff}}}\,. \tag{4.6}$$

The effective Hagedorn temperature depends on the string scale and hence on a phase in the theory. However a genuine Hagedorn temperature $\mathcal{T}_H$ is uniquely determined by the minimal scale, where a transition leads to the Hagedorn phase. Figure 1, in section 4.3, shows a schematic demonstration of the Hagedorn transition in presence of an E-field. The effective string tension obtained from eq. (2.13) can be appropriately expressed in terms of temperature using eq. (4.6). Then the string tension becomes

$$T_{\rm eff} = T\left(\frac{\mathcal{T}_H^2}{\mathcal{T}^2} - 1\right)^{1/2}. \tag{4.7}$$

Thus, the gravity decoupling limit, in thermal string descrption, is described by $\mathcal{T}\to\mathcal{T}_H$.

4.2 Critical regime: gauge-string(s) decoupling

In this section, we shall notice that the Hagedorn limit $\mathcal{T}\to\mathcal{T}_H$ does not correspond to a single sharp transition, rather it is associated with a series of transitions in a critical regime.

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In fact, the phase transition begins in the gravity decoupling limit, passes through a large number of string phases and leads to the Hagedorn phase. Let us qualify the statement by recalling the Hagedorn transition in a fundamental string theory [26]. The mass-less closed string backgrounds are the metric, an arbitrary two-form and a dilaton. At this point, we recall the U(1) gauge invariant combination $\bar{\mathcal{F}} = (b + \bar{F})$ for an arbitrary b-field in the theory. The b-field and the gauge field belong to the mass-less spectrum of closed and open string respectively. Taking account for the mass-less scalar (in a higher order in α') and/or a mass-less pseudo-scalar (dual of two-form) the free energy eq. (4.5) can be seen to be lowered in the decoupling limit. In other words, the transition takes place at a critical temperature below the Hagedorn temperature. A first order phase transition has been shown to take place at a temperature, just below $\mathcal{T}_H$.

In the present context, the two-form b-field is a global mode of the theory. It decouples from the bulk dynamics of the closed string and couples to the open string boundary dynamics. Though an arbitrary b-field in the theory has its origin in the mass-less closed string spectrum, a constant b-field does not belong to the closed string spectrum any more, rather it is associated with the U(1) gauge interaction at the string boundary. The global modes of b-field, modifies the U(1) gauge invariance to a non-local one in the theory. The spontaneous breaking of the local symmetry gives mass-term to the gauge field and the b-field is absorbed in an enhanced U(1) symmetry i.e. a noncommutative U(1) symmetry [12]. The non-local extension is due to the non-linearity in EM-field and gives rise to gauge-string description in the critical regime. Taking into account the non-zero mass gauge/string windings in eqs(3.9) and (3.18), the effective critical (Hagedorn) temperature (4.5) is lowered than the intrinsic Hagedorn temperature. As a result, the free energy is lowered and the transition can be seen to be associated with a discontinuity in free energy and describes a first order transition. All the closed string windings are favoured for decoupling due to their vanishing mass during the transition.

Subsequently, as the temperature is increased in the regime ($\mathcal{T} \rightarrow \mathcal{T}_H$), at a certain critical temperature T_H^{eff}, it overcomes the threshhold and a fraction of non-zero mass string/gauge windings are decoupled from the theory. Since the effective string scale $\alpha'_{\text{eff}} = n\alpha'$ is quantized in the unit of "rigid" scale α', the string scale in the effective description is lowered $(\alpha'_{\text{eff}})^{\text{new}} = (n-m)\alpha' < \alpha'_{\text{eff}}$. It implies that the critical (Hagedorn) temperature (4.6) in the theory increases in each step, with the decoupling of light string from the gauge field in the theory. In other words, the critical temperature $\mathcal{T}_H^{\text{eff}}$ is self-tuned along with the decoupling of gauge-strings at successive stages and the genuine Hagedorn transition recedes.

In order to analyze the nature of phase transition, we consider the partition function for the thermal string in the critical regime. The hamiltonian in the regime can be obtained from eq. (4.4) and the partition function becomes

$$\mathcal{Z} = \text{Tr} \exp \left(-\beta L_0^b \right) . \tag{4.8}$$

In the regime, the interaction hamiltonian plays a leading role due to significant contribution from the light string modes. Explicitly, the expression (in the limit $\mathcal{T} \rightarrow \mathcal{T}_H$ from below) becomes

$$\mathcal{Z} = \mathcal{Z}_0 \mathrm{Tr}\exp\left(-\beta_H L_0^w\right) , \tag{4.9}$$

where $\mathcal{Z}_0$ denotes the partition function in absence of EM-field. L_0^w denotes the interaction hamiltonian, which can be obtained from eq. (4.4). In the regime, the string energy is significantly large and β_H is "fixed". The expression (4.9) describing the sum over gauge-string states, converges and the total free energy $\mathcal{E} = -\mathcal{T}\ \ln\mathcal{Z}$ is analytic during the transitions. However, the specific heat $\mathcal{E}/(\mathcal{T}-\mathcal{T}_H)$ shows non-analytic behaviour in the critical regime and confirms a second order transition. A series of discrete transitions of similar kind, have been shown to be second order due to a large number of closely spaced first order transitions [27, 28]. Interestingly, in these papers, the second order phase transitions have been discussed, with an appropriate thermal ensemble. There, the decoupling of strings at the transition, have been shown from the bound state of $D(p)$-branes with F-strings in type-II string theories.

The critical regime ($\mathcal{T}\ \to\mathcal{T}_H$), is descibed by the phase area ABC under the curve depicted in figure 2, in section 4.3. Various phases there, correspond to a thermal string, which comprises NCOS phase at its bottom (along BA). The mixing of time with space components from the momentum modes begins in NCOS phase. The mixing is reflected due to the presence of small mass (string $\leftrightarrow$ non-linear gauge) excitations in the phase. With an increase in temperature, i.e. along $B\to C$ in the critical regime, the new light strings begin to decouple from the theory. The fact that time dimension is small and the space-dimensions are large, imply a large winding number for time than the space coordinates. Thus the decoupled gauge-strings, during the second order phase transition, comprises a significant number of time-like windings than the space-like ones. In the case, a mixed string state may be approximated by that of time-like ones. Intuitively, the decoupling of time-like string modes, during a series of transitions, partly drains off the non-linearity in the gauge field. In this context, let us consider the $D(3)$-brane dynamics (2.11) and recall some of the relevant mappings, established in the literatures, to gain some insight on the high temperature phase in the theory.

In the effective description, the DBI lagrangian density (2.11) can be expressed as that for a noncommutative U(1) to a leading order in α'. In the limit $\alpha'\to 0$, the $D(3)$-brane dynamics reduces to U(1) gauge theory [12] with $\star$-product in its definition. Schematically, $\mathcal{L}_{\mathrm{DBI}} \to \mathcal{L}^{nc}_{\mathrm{DBI}}$ the noncommutative DBI lagrangian density, which can be expressed as

$$\mathcal{L}^{nc}_{\mathrm{DBI}} = \frac{1}{8\pi G_s}\sqrt{\det G}G^{\mu\lambda}G^{\nu\rho} Tr F^{nc}_{\mu\nu}\star F^{nc}_{\lambda\rho} + \text{total derivatives}\,. \tag{4.10}$$

Taking account for the temporal noncommutativity, the non-linear relation between the field strength for a noncommutative U(1) field and that for (ordinary) gauge field can be generalized from ref. [12]. To first order in parameter Θ ($=\Theta_E$ and Θ_B), the noncommutative field strength becomes

$$F^{nc}_{\mu\nu} = F_{\mu\nu} + \Theta^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho} - \Theta^{\lambda\rho}A_\lambda\partial_\rho F_{\mu\nu}\,. \tag{4.11}$$

Now the noncommutative DBI lagrangian $\mathcal{L}^{nc}_{\rm DBI}$ can be expressed in terms of non-commutative U(1) lagrangian density with stringy corrections.

$$\mathcal{L}^{nc}_{\rm DBI} = -\frac{1}{4}\Big(F^{nc}\Big)^2 + \mathcal{O}(\alpha')\,, \tag{4.12}$$

$$\text{where} \quad -\frac{1}{4}\Big(F^{nc}\Big)^2 = \frac{1}{2}\Big(E^2 - B^2\Big)\Big(1 + \Theta_E \cdot E + \Theta_B \cdot B\Big) - \\ -\Big(E \cdot B\Big)\Big(\Theta_E \cdot B + \Theta_B \cdot E\Big)\,. \tag{4.13}$$

Since gravity is decoupled from the theory, knowing noncommutative scale shall determine the stringy scale in the theory. The non-linearity in U(1) theory can be obtained from eq. (4.13). Interestingly, the trace of energy-momentum tensor T^{μ}_{μ} calculated in the gauge theory can be seen to be equal to the required non-linearity in the dynamics. To $\mathcal{O}(\Theta)$, the the trace becomes

$$T^{\mu}_{\mu} = \left(E^2 - B^2\right) g - (E \cdot B)\left(\frac{\alpha'}{\alpha'_{\perp}} + \frac{\alpha'_{\perp}}{\alpha'}\right)_{\rm eff}\,. \tag{4.14}$$

In the effective description, the EM-string corrections $\mathcal{O}(\alpha')$ cancel the gauge non-linearity and describes a unitary theory. For $E \perp B$, the non-linearity in the U(1) gauge theory can be simplified to obtain $g|E|^2$ and for the parallel field configuration, it becomes $-2g|E|^2$ (or $-2g|B|^2$). It implies that the non-linearity in the U(1) gauge theory is entangled with the EM-field. Figure 1 in section 4.3, shows that the gauge non-linearity is maximum at A, i.e. in the beginning of the critical regime. It becomes feeble with the decoupling of gauge-strings and ultimately takes a minimal value at $\mathcal{T}_H$. A careful analysis at this point suggests that the entanglement possesses its grain in the global modes of b-field. At a minimal non-linearity, the b-field forms a condensate $\langle T^{\mu}_{\mu}\rangle$, which is schematically shown along the curve CD in figure 2. As a consequence, the Hagedorn phase becomes free from the gauge non-linearity and tend to describe an ordinary gauge theory $F^{nc}{\rightarrow}F$. In the effective string description, the noncommutative U(1) symmetry is spontaneously broken to the U(1) gauge symmetry, by giving a vacuum expectation value to the b-field at $\mathcal{T}_H$. The phenomenon is analogous to that of deconfinement in QCD. In presence of a large number (N) of coincident branes, the gauge group becomes U(N) and the theory describes a noncommutative U(N) symmetry. There, the Hagedorn phase becomes subtle due to the presence of additional (non-abelian gauge) symmetry. Presumably, a spontaneous breaking of noncommutative symmetry, followed by that of non-abelian symmetry [22] leads to the deconfinement in QCD.

4.3 Hagedorn phase: gauge-string condensate and tachyon decoupling

In this section, we re-investigate the critical limit $\mathcal{T}{\rightarrow}\mathcal{T}_H$ carefully to explain some of the important issues at the phase transition. To be specific, the limit can be viewed as the one from below (left) i.e. $(\mathcal{T} - \mathcal{T}_H){\rightarrow}0^-$ as well as the one from above (right) i.e. $(\mathcal{T} - \mathcal{T}_H){\rightarrow}0^+$. In the (left) limit, $\alpha'{\rightarrow}\alpha'_{\rm eff}$, i.e. the electric string description can be replaced by that of an effective thermal string. On the other hand, in the (right) limit the string scale becomes

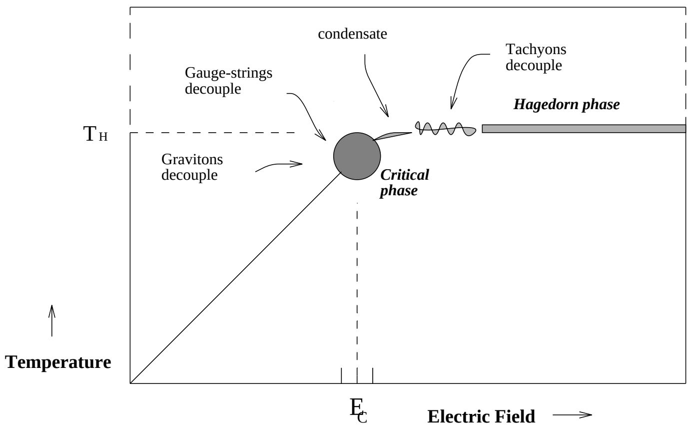

Figure 1: *Notion of temperature in presence of an EM-field.* A time-like noncommutative string description can be equivalently described by that of thermal string. Various phases are obtained after decoupling all the gravitons, gauge-strings, their condensate and tachyons.

$\alpha' \to -\alpha'_{\rm eff}$. Then, the thermal string mode expansion can be obtained from eq. (4.1). It becomes

$$X^0 = x^0 + \frac{(4\pi\alpha'_{\rm eff})m'}{\beta}\tau + \frac{m\beta}{\pi}\sigma - \sqrt{2\alpha'_{\rm eff}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(\alpha_n^0\ \cos n\sigma + i\alpha_n^1 \sin n\sigma\right) . \qquad (4.15)$$

The constant momentum and winding modes, in the regime, remain unchanged from that of left limit. However the oscillator sector has undergone an exchange between the imaginary and real components. Then an effective string state with mass M_s satisfies

$$-M_s^2 = \frac{N}{\alpha'_{\rm eff}} . \qquad (4.16)$$

It implies that the effective gauge-string have experienced a critical transition, i.e. $\alpha'_{\rm eff} \leftrightarrow -\alpha'_{\rm eff}$, at $\mathcal{T}_H$. An extrapolation for $\alpha'_{\rm eff}$ around $\mathcal{T}_H$ i.e. from below to above and vice-versa, assures $\alpha'_{\rm eff}\to 0$ at $\mathcal{T}_H$. Since $\alpha'_{\rm eff} = (n-m)\alpha'$, such a phase is associated with the decoupling of a large number of gauge-strings (large m), just below $\mathcal{T}_H$. It leads to the condensation $\langle T^\mu_\mu\rangle$ of gauge-string in the effective description at $\mathcal{T}_H$. Thus, the gauge-string becomes heavy $\mathcal{T}_{\rm eff}\to$large at $\mathcal{T}_H$ and decouples from the phase. It is followed by the decoupling of tachyonic particles/electric dipoles at $\mathcal{T}_H$ and leads to the Hagedorn phase. The total energy at $\mathcal{T}_H$ can be given by

$$\mathcal{E} = \left(T_{\rm eff} + \frac{N}{\alpha'_{\rm eff}}\right) l_s . \qquad (4.17)$$

For minimum energy $d\mathcal{E}/dl_s = 0$. Then, a stable configuration is described by $T_{\rm eff} = M_s^2$. Thus an increase in energy at $\mathcal{T}_H$, excites the minimal scale to a large string scale, which becomes unstable and ultimately decays to a stable phase consisting of non-interacting gauge particles.

Now the partition function, in the limit of $\mathcal{T}\to\mathcal{T}_H$ from above, can be obtained from eq. (4.9) by accounting the correct expression for the hamiltonian there. It can be expressed

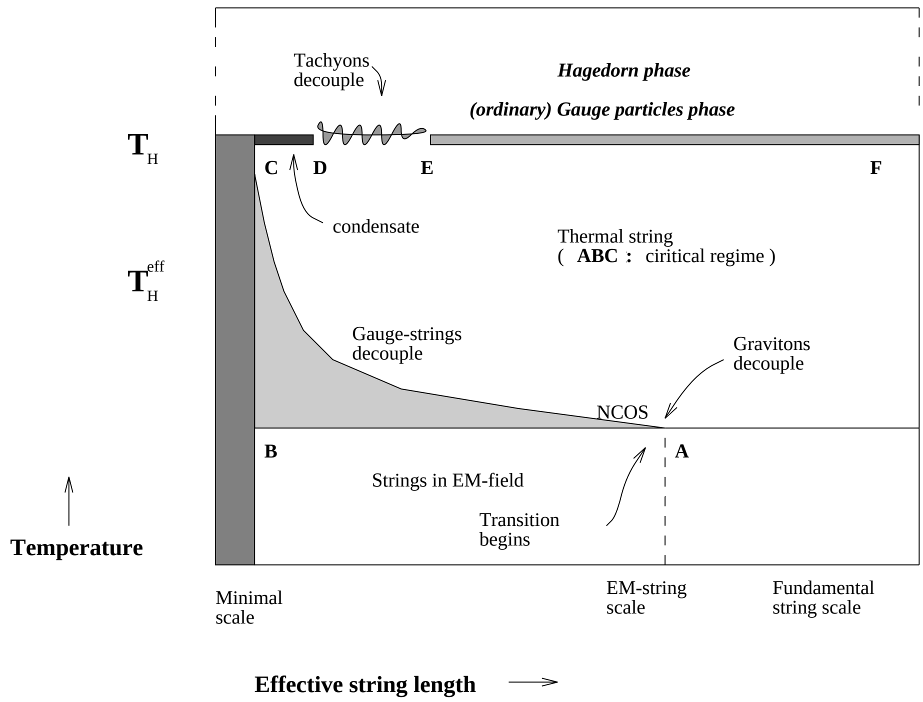

Figure 2: *Critical regime leading to Hagedorn phase.* Phases involved in an effective thermal string description is shown by considering the variation of string length with temperature. The area ABC describes the critical regime for the thermal string. A first order phase transition is shown along BA, where gravity is completely decoupled to yield a NCOS phase. As an immediate feature, a series of second order phase transitions, at respective $\mathcal{T}_H^{\rm eff}$, involving gauge-strings is shown from BA to C. In the limit of a critical temperature, the new phase is described by a relatively small string scale and hence the decoupled gauge-strings are "long" strings in the theory. A further increase in energy at $\mathcal{T}_H$, increases the gauge-string scale and describes the "long" string phase (CF). Such a "long" gauge-string phase is unstable and decays to (ordinary) gauge particles (EF) by releasing excess energy in the guise of "long" strings (CD + DE). The gauge-strings condensate (CD), before they become tachyonic (DE). The flip between the notions of time and temperature may be summarized in a table:

Phase:	**Time**	**Temperature**
Below AB:	valid	valid
Area ABC:	breaks	valid
Along EF:	valid	breaks

as

$$\mathcal{Z} = \mathcal{Z}_0 \mathrm{Tr} \exp\left(\beta_H L_0^w\right) . \tag{4.18}$$

The non-convergence of the partition function implies its break down. Then, the free energy $\mathcal{E} = -\mathcal{T} \ln \mathcal{Z}$ at $\mathcal{T}_H$ shows a logarithmic divergence. Thus, the Hagedorn phase is described by a high density of states at $\mathcal{T}_H$. A schematic details on the Hagedorn transition(s), leading to the Hagedorn phase is described in figure 2.

At this point, we recall the time translation symmetry in the critical regime to analyze the exchange of notion between time and temperature. By now, it is certain that the notion of time breaks down in the critical regime. However at the expense of time, the notion of temperature becomes inevitable in the regime. Thus, the $(3+1)$-dimensional space-time is described by a non-simplectic manifold $R^3 \times S^1$ in the critical regime. Since a tree level world-sheet can not be described by $R^3 \times S^1$ topology, it confirms that the free energy should begin at one-loop order in the regime. However at $\mathcal{T}_H$, the free energy can be estimated from eq. (4.5). Computation of free energy for the thermal string [26], shows $1/g^2$ dependence at $\mathcal{T}_H$, where g denotes the genus of the world-sheet. It provides an evidence for the tree level world-sheet contribution to the free energy at $\mathcal{T}_H$. Since the tree-level world-sheet is simply connected, the $R^3 \times S^1$ does not remain valid in the Hagedorn phase. The time translational symmetry is lost there, leading to a break down of temperature. The lost notion of time, in the critical regime, is regained in the Hagedorn phase.

5. Concluding remarks

To conclude, we have considered the propagation of open bosonic string, in presence of a constant two-form b-field. Under appropriate Dirichlet boundary conditions, the string dynamics reduces to that of a $D(3)$-brane in a constant EM-field. Incorporating different orientations of a B-field with respect to the E-field, we have performed an analysis, to explain some of the phenomena associated with the new light string modes in a gravity decoupled theory. We emphasized on the presence of space and time-like windings in the $(3+1)$-dimensional theory and explained the fate of lorentzian signature of space-time there. Taking account for a inherent relation between the space and time-like windings, we have obtained bounds on the winding radii. It is shown that the time-like coordinate is bounded by the string scale and describes a small dimension in the theory. As a result, the space coordinates in the theory are argued to be large in comparison to the time dimension. The small time-like coordinate implies discrete notion of time in the EM-string theory. For a large number of closely spaced time intervals, discrete notion may be approximated to a continuous one. However, the continuous parameter does not evolve with time and results in a time translation symmetry in $(3+1)$-dimensions. It resolves the apparent paradox of compact time in a lorentzian signature of space-time. In other words, the notion of time breaks down in the gravity decoupling limit and at the expense, temperature becomes significant there. The phenomenon is supported by a non-trivial mixing of time with the spatial components of light gauge-string modes.

In addition, the presence of gauge field windings in the theory is emphasized and it is argued that the the effective string scale is quantized in the units of "rigid" string which may be interpreted as an electric dipole. Our analysis suggests that the effective string is made from a large number of electric dipoles aligned along the direction of E-field. In addition, the dipoles joined end to end in a series, since $T_{\text{eff}} \to 0$. In other words, a large space-like dimension gives rise to the "long" string description in the effective theory. In fact the winding modes give rise to non-locality, which is an integral part of non-linearity in gauge field.

One of the important lesson, we learned, is that the EM-field has many interesting facets in a open string theory. On the one hand, it describes the string dynamics on a noncommutative space-time and on the other, it introduces the notion of thermal ensemble into a theory. We have performed the thermal string mode analysis to explicitly account for the winding modes in the theory. The mass-shell condition for the thermal strings is analyzed and we have obtained an expression for the critical (effective Hagedorn) temperature in the theory. The critical regime ($\mathcal{T}\rightarrow\mathcal{T}_H$), described by the area ABC in figure 2, is investigated for transitions leading to the Hagedorn phase. Taking into account the small/large dimensions, we have argued that the gauge-string phase is dominated by time-like windings in the theory.

In the critical regime, we have obtained the corrections to (ordinary) U(1) gauge theory, due to its own non-linearity. The gauge/string corrections are due to the constant b-field in the theory. At $\mathcal{T}_H$, a spontaneous break down of noncommutative U(1) symmetry is argued, analogus to that in QCD at its deconfinement temperature. The symmetry breaking gives a mass to the b-field and forms condensate at $\mathcal{T}_H$. In addition, we have shown the presence of tachyonic windings in the "long" string phase. It is argued that a further increase in energy at $\mathcal{T}_H$, excites the string and decouples the tachyons from the phase. Then, the Hagedorn phase is described by the ordinary (non-interacting) gauge particles. In the context, we have out-lined a generalization to noncommutative U(N) theory, in presence of N number of coincident $D(3)$-branes. It will be interesting to analyze the critical regime there, to work out the details of phase transitions leading to the Hagedorn phase.

In particular, the new windings, share some common properties between the finite temperature [34] and space-time noncommutative field and string theories. The fact that the space and time-like windings naturally determine the large and small dimensions in a theory, may shed some light on the low-scale gravity models.

On the other hand, the gauge-string theory is a subject by itself and its study may provide important tools to address some of the conceptual issues in quantum gravity with various alternate effective descriptions. For example, the genus zero $g=0$ contribution of gauge-string to the free-energy, possibly, requires substantial correction to riemannian manifold at short distances. It urges a non-perturbative formulation and is beyond the scope of this paper.

Acknowledgments

Author (S.K.) is grateful to S.D. Joglekar and A. Sen for useful discussions, at the early stage of this work. He thanks P. Jain and K. Ray for various discussions, in general, on the topic. Some of the results obtained in this article, were presented by him in Indo-Russian satellite workshop on String theory and Integrable models, 22-23 January 2002 at Harish Chandra Research Institute, Allahabad.

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